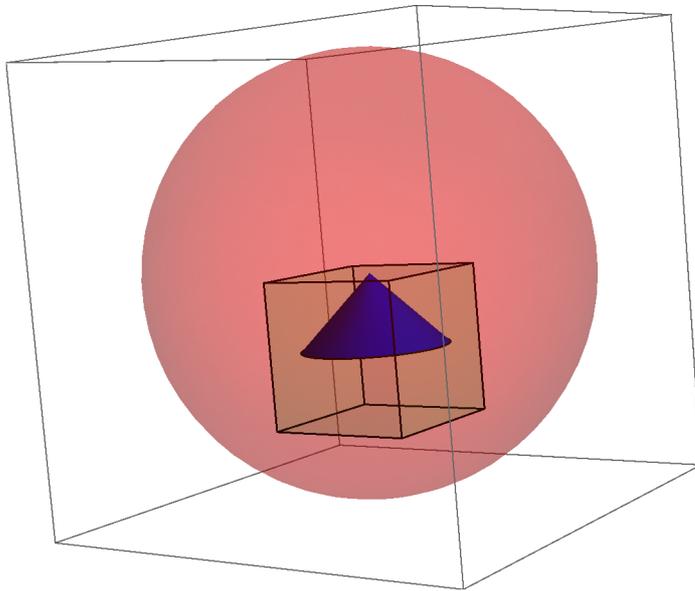


d

Plotting Surfaces

I. Built in Surfaces

```
Graphics3D[{Red, Opacity[0.3], Sphere[{0, 0, 1}, 3],  
  Opacity[1], Blue, Cone[{0, 0, 0}, {0, 0, 1}], 1},  
  Green, Opacity[0.1], Cuboid[{-1, -1, -1}, {1, 1, 1}]]]
```



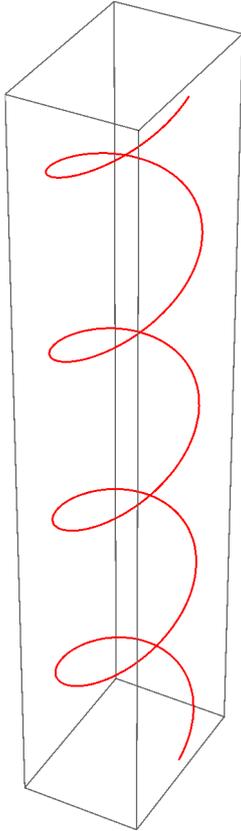
2. Tubes

? Tube

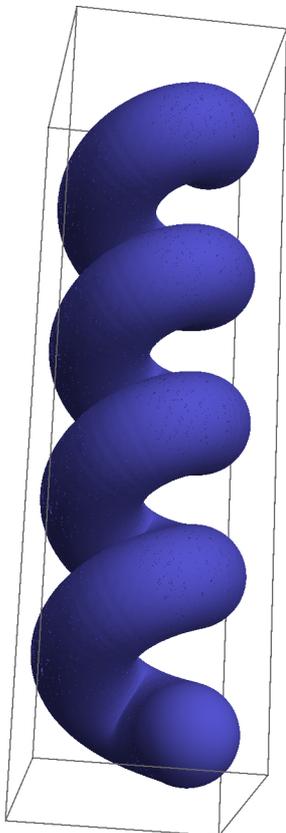
Tube[{{x₁, y₁, z₁}, {x₂, y₂, z₂}, ...]} represents a 3D tube around the line joining a sequence of points.
Tube[{pt₁, pt₂, ...}, r] represents a tube of radius *r*.
Tube[{{pt₁₁, pt₁₂, ...}, {pt₂₁, ...}, ...]} represents a collection of tubes.
Tube[*curve*, ...]} represents a tube around the specified 3D curve. >>

```
r[t_] := {2 Cos[t], 3 Sin[t], t};
```

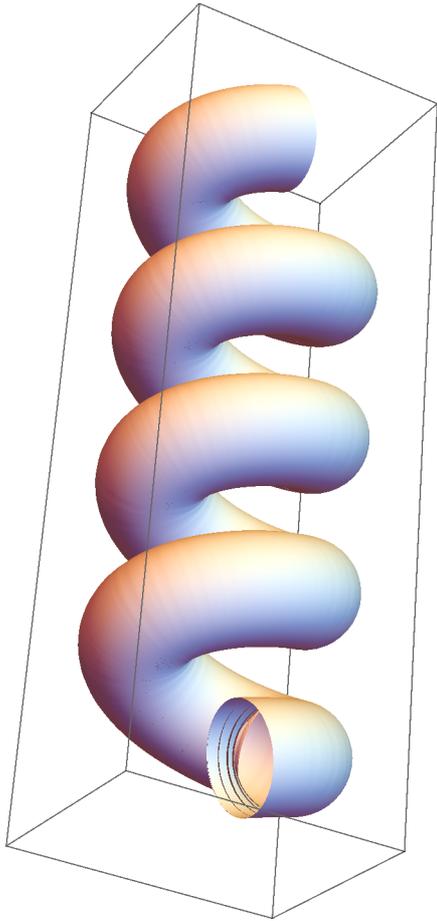
```
helix = ParametricPlot3D[r[t], {t, -4 Pi, 4 Pi},  
  ColorFunction -> Function[{x, y, z}, RGBColor[1, 0, 0]], Axes -> False]
```



```
Show[Normal[helix] /. Line[x_>] -> Tube[x, 2], PlotRange -> All]
```



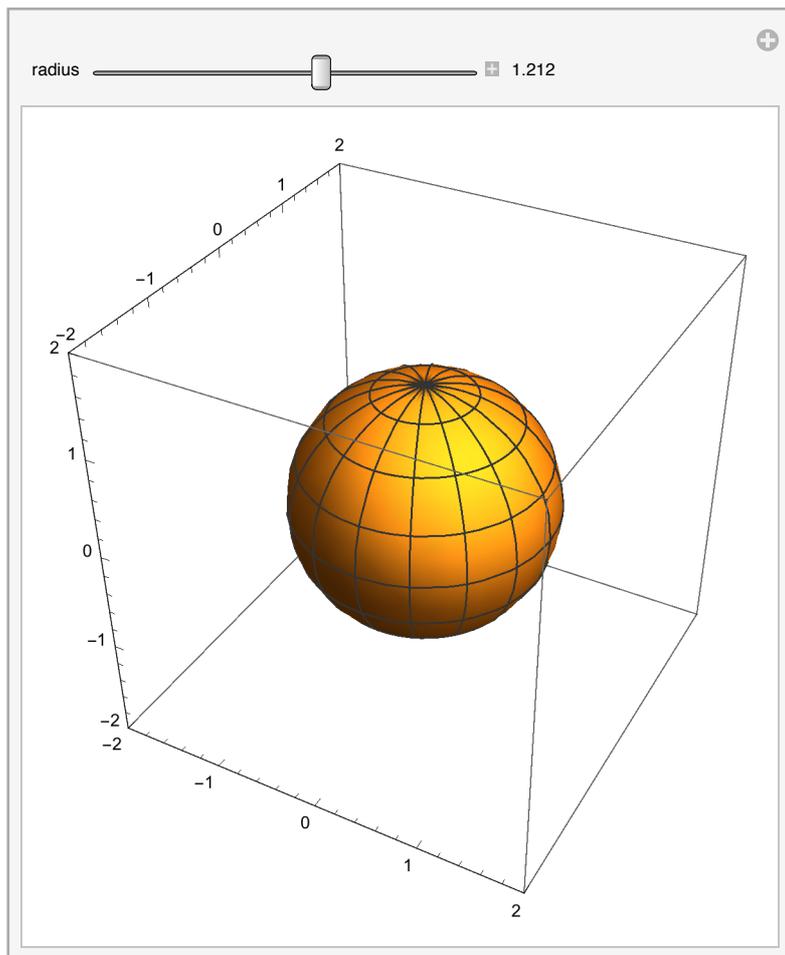
```
Graphics3D[{CapForm[None], Tube[Normal[helix][[1, 1, 3, 2]], 2]}]
```



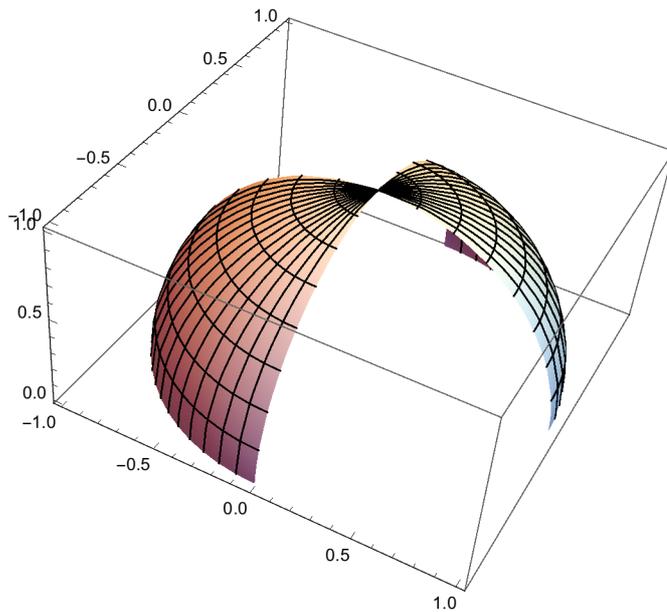
3. ParametricPlot3D

Sphere

```
Manipulate[ParametricPlot3D[r {Cos[ $\theta$ ] Cos[ $\phi$ ], Cos[ $\phi$ ] Sin[ $\theta$ ], Sin[ $\phi$ ]},  
  { $\theta$ , 0, 2 Pi}, { $\phi$ , 0, 2 Pi}, PlotRange -> {{-2, 2}, {-2, 2}, {-2, 2}},  
  {{r, 1, "radius"}, 0.001, 2, Appearance -> "Labeled"}]
```



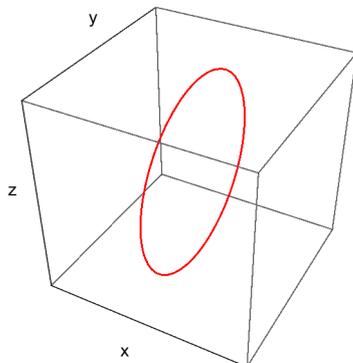
```
ParametricPlot3D[{Cos[θ] Cos[φ], Cos[φ] Sin[θ], Sin[φ]}, {θ, 0,  $\frac{\text{Pi}}$ }, {φ, 0, Pi}]
```



Torus

```
circle[θ_] := {0, Sin[θ], Cos[θ] + 4}
```

```
ParametricPlot3D[circle[t], {t, -Pi, Pi}, ColorFunction -> (Red &),  
AxesLabel -> {"x", "y", "z"}, ImageSize -> Small, Ticks -> False]
```



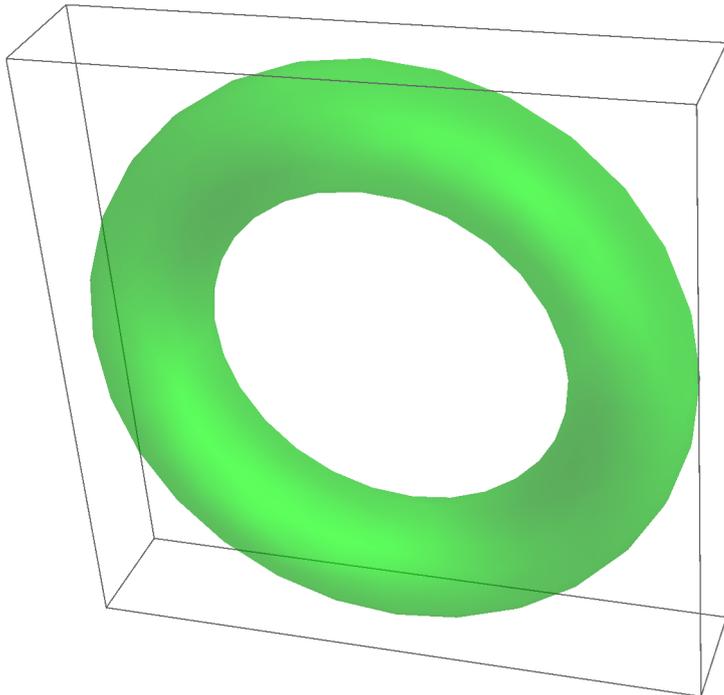
```
Clear[tor]
```

```
tor[θ_, φ_] := RotationTransform[φ, {0, 1, 0}][circle[θ]]
```

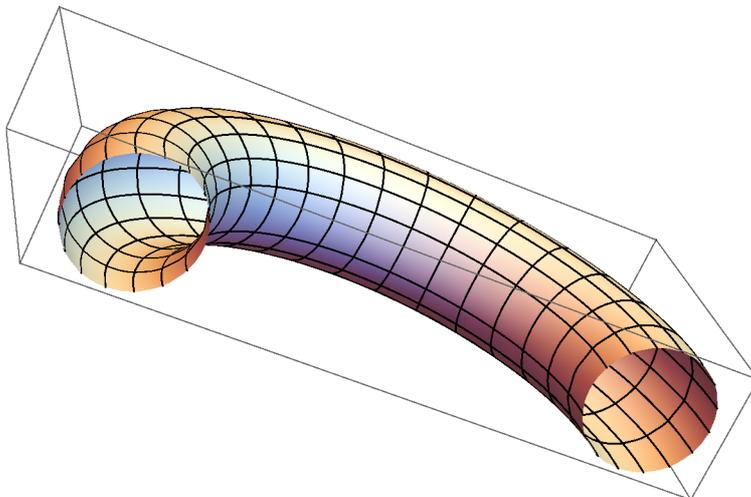
```
tor[θ, φ]
```

```
{(4 + Cos[θ]) Sin[φ], Sin[θ], (4 + Cos[θ]) Cos[φ]}
```

```
tr = ParametricPlot3D[tor[ $\theta$ ,  $\phi$ ], { $\theta$ , -Pi, Pi}, { $\phi$ , -Pi, Pi}, Axes → False,
  Mesh → None, ColorFunction → (Directive[Opacity[0.4], Green] &)]
```

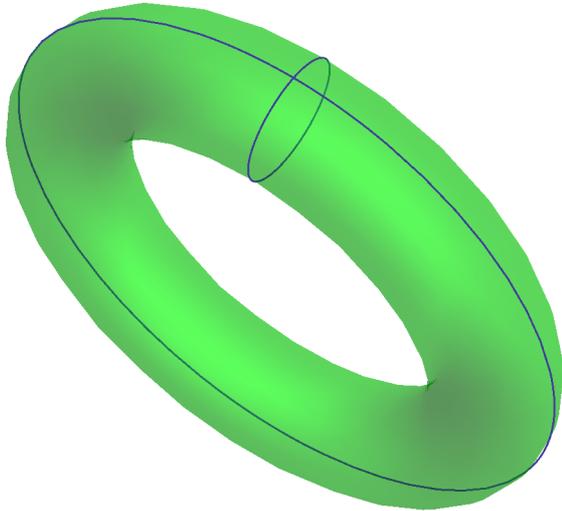


```
ParametricPlot3D[tor[ $\theta$ ,  $\phi$ ], { $\theta$ , -Pi, Pi}, { $\phi$ , -Pi/2, Pi/2}, Axes → False]
```



```
 $\phi$ axis =
  ParametricPlot3D[Evaluate[tor[0,  $\phi$ ]], { $\phi$ , -Pi,  $\pi$ }, Boxed → False, Axes → False];
 $\theta$ axis =
  ParametricPlot3D[Evaluate[tor[ $\theta$ , 0]], { $\theta$ , -Pi,  $\pi$ }, Boxed → False, Axes → False];
```

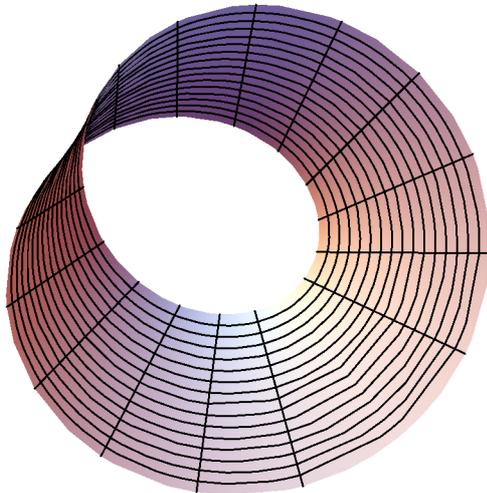
```
Show[tr,  $\theta$ axis,  $\phi$ axis, Boxed  $\rightarrow$  False, Axes  $\rightarrow$  False, ImageSize  $\rightarrow$  Small]
```



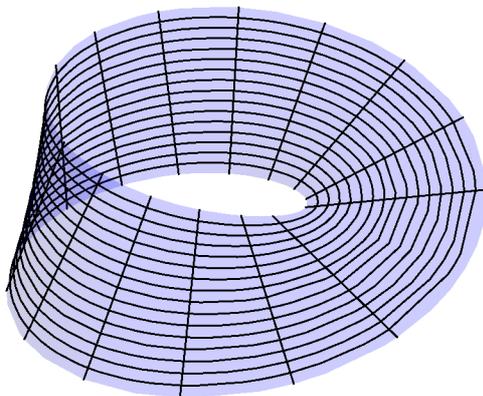
Moebius Strip

```
moebiusstrip[u_, v_] := {Cos[u] + v Cos[u / 2] Cos[u],  
Sin[u] + v Cos[u / 2] Sin[u], v Sin[u / 2]}
```

```
ParametricPlot3D[moebiusstrip[u, v],  
{u, 0, 2 Pi}, {v, -0.5, 0.5},  
Axes -> None, Boxed -> False]
```

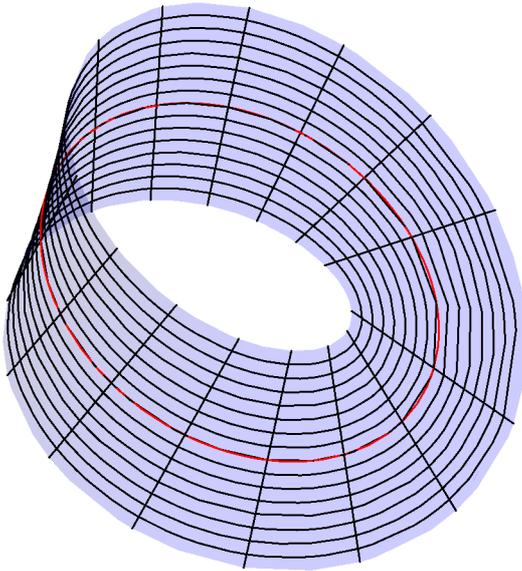


```
mb = ParametricPlot3D[moebiusstrip[u, v],  
{u, 0, 2 Pi}, {v, -0.5, 0.5},  
Axes -> None, Boxed -> False,  
ColorFunction -> Function[{x, y, z}, Directive[Opacity[0.2], Blue]]]
```



```
mb1 = ParametricPlot3D[moebiusstrip[u, 0],  
{u, 0, 2 Pi},  
Axes -> None, Boxed -> False, ColorFunction -> Function[{x, y, z}, Red]];
```

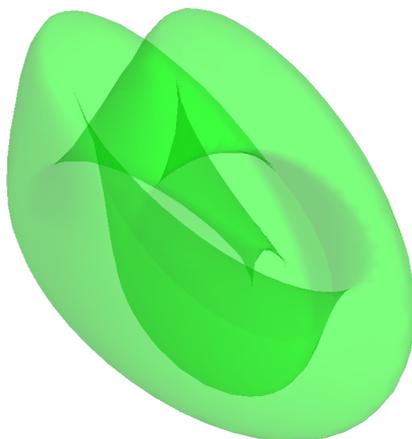
```
Show[mb, mb1]
```



Klein Bottle

```
klein1[θ_, φ_] := { (2 + Cos[φ / 2] Sin[θ] - Sin[φ / 2] Sin[2 θ]) Cos[φ],
  (2 + Cos[φ / 2] Sin[θ] - Sin[φ / 2] Sin[2 θ]) Sin[φ],
  Sin[φ / 2] Sin[θ] + Cos[φ / 2] Sin[2 θ] }
```

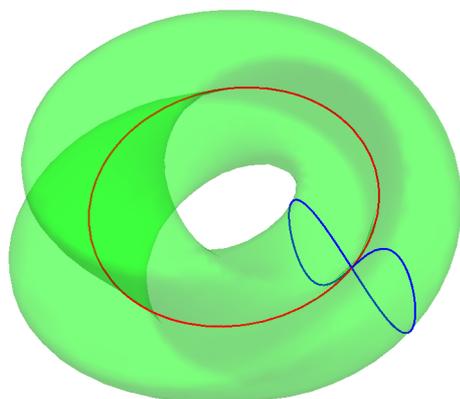
```
k11 = ParametricPlot3D[klein1[θ, φ], {θ, -Pi, Pi}, {φ, -Pi, Pi}, Boxed → False,
  Axes → False, ColorFunction → (Directive[Opacity[0.3], Green] &), Mesh → False]
```



```

φaxis = ParametricPlot3D[klein1[0, φ], {φ, -Pi, Pi}, ColorFunction -> (Red &)];
θaxis = ParametricPlot3D[klein1[θ, 0], {θ, -Pi, Pi}, ColorFunction -> (Blue &)];
Show[kl1, φaxis, θaxis, PlotRange -> All, Boxed -> False, Axes -> False]

```



```

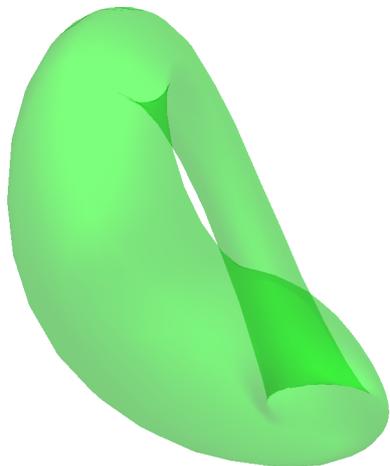
klein2[u_, v_] := With[{bx = 6 Cos[u] (1 + Sin[u]),
  by = 16 Sin[u], rad = 4 (1 - Cos[u] / 2)}, {rad Sin[v],
  Piecewise[{{bx + rad Cos[v + Pi], Pi < u <= 2 Pi}}, bx + rad Cos[u] Cos[v]],
  Piecewise[{{by, Pi < u <= 2 Pi}}, by + rad Sin[u] Cos[v]]]}]

```

```

k12 = ParametricPlot3D[klein2[ $\theta$ ,  $\phi$ ], { $\theta$ , -Pi, Pi}, { $\phi$ , -Pi, Pi}, Boxed  $\rightarrow$  False,
  Axes  $\rightarrow$  False, ColorFunction  $\rightarrow$  (Directive[Opacity[0.3], Green] &), Mesh  $\rightarrow$  False]

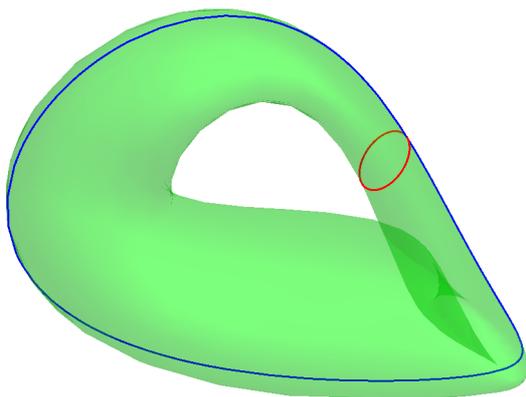
```



```

 $\phi$ axis2 = ParametricPlot3D[klein2[0,  $\phi$ ], { $\phi$ , -Pi, Pi}, ColorFunction  $\rightarrow$  (Red &)];
 $\theta$ axis2 = ParametricPlot3D[klein2[ $\theta$ , 0], { $\theta$ , -Pi, Pi}, ColorFunction  $\rightarrow$  (Blue &)];
Show[k12,  $\phi$ axis2,  $\theta$ axis2, PlotRange  $\rightarrow$  All, Boxed  $\rightarrow$  False, Axes  $\rightarrow$  False]

```



The Projective Plane

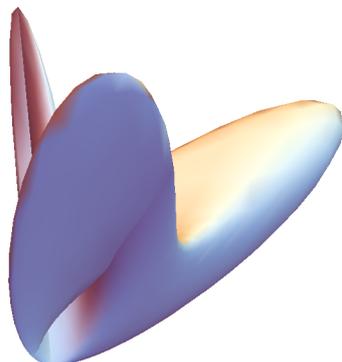
```

boySurface[a_][u_, v_] := a {

$$\frac{\sqrt{2} \cos[v]^2 \cos[2u] + \cos[u] \sin[2v]}{2 - \sqrt{2} \sin[3u] \sin[2v]},$$


$$\frac{\sqrt{2} \cos[v]^2 \sin[2u] + \cos[u] \sin[2v]}{2 - \sqrt{2} \sin[3u] \sin[2v]}, \frac{3 \cos[v]^2}{2 - \sqrt{2} \sin[3u] \sin[2v]}}
ParametricPlot3D[boySurface[1][u, v], {u, -Pi/2, Pi/2},
{v, 0, Pi}, Mesh -> False, Axes -> False, Boxed -> False, PlotRange -> All]$$

```

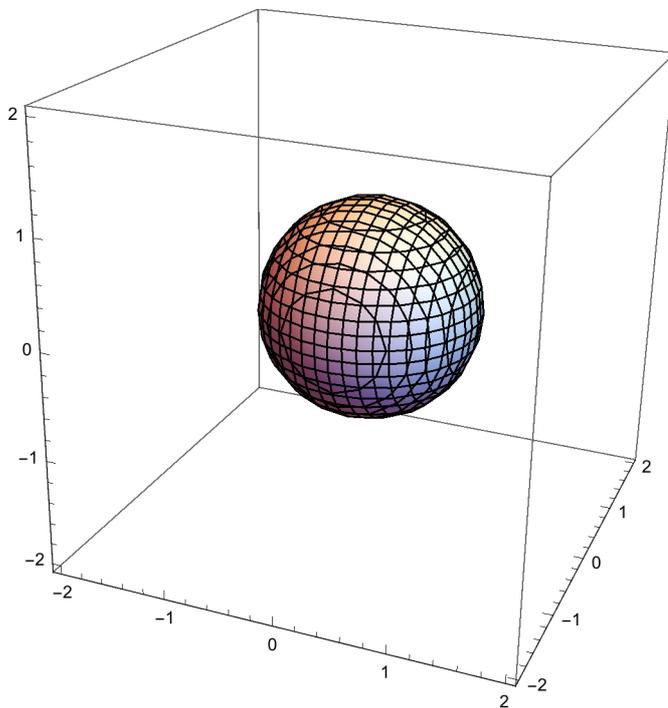


Roman Surface

4. ContourPlot3D

Sphere

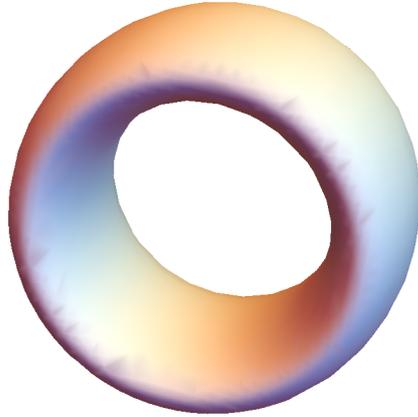
```
ContourPlot3D[x2 + y2 + z2 == 1, {x, -2, 2}, {y, -2, 2}, {z, -2, 2}]
```



Torus

```
cartesianTorus[x_, y_, z_] :=  
x4 + 2 y2 x2 + 2 z2 x2 - 34 x2 + y4 + z4 + 30 y2 + 2 y2 z2 - 34 z2 + 225
```

```
ContourPlot3D[cartesianTorus[x, y, z] == 0, {x, -6, 6},  
{y, -2, 2}, {z, -6, 6}, Mesh → False, Boxed → False, Axes → False]
```



```
cartesianDoubleTorus[x_, y_, z_] := - $\frac{1}{2} + x^2 + y^4 - 2y^2z^2 + z^4 + \frac{y^2z^4}{2} - \frac{z^6}{2} + \frac{z^8}{16}$ 
```

```
ContourPlot3D[cartesianDoubleTorus[x, y, z] == 0, {x, -3, 3},  
{y, -3, 3}, {z, -3, 3}, Mesh → False, Boxed → False, Axes → False]
```



```

cartesianKlein[x_, y_, z_] :=
768 x4 - 1024 x5 - 128 x6 + 512 x7 - 80 x8 - 64 x9 + 16 x10 + 144 x2 y2 - 768 x3 y2 - 136 x4 y2 +
896 x5 y2 - 183 x6 y2 - 176 x7 y2 + 52 x8 y2 + 400 y4 + 256 x y4 - 912 x2 y4 + 256 x3 y4 +
315 x4 y4 - 144 x5 y4 - 16 x6 y4 + 4 x8 y4 - 904 y6 - 128 x y6 + 859 x2 y6 - 16 x3 y6 -
200 x4 y6 + 16 x6 y6 + 441 y8 + 16 x y8 - 224 x2 y8 + 24 x4 y8 - 76 y10 + 16 x2 y10 + 4 y12 -
2784 x3 y z + 4112 x4 y z - 968 x5 y z - 836 x6 y z + 416 x7 y z - 48 x8 y z + 1312 x y3 z +
2976 x2 y3 z - 5008 x3 y3 z - 12 x4 y3 z + 2016 x5 y3 z - 616 x6 y3 z - 64 x7 y3 z + 32 x8 y3 z -
1136 y5 z - 4040 x y5 z + 2484 x2 y5 z + 2784 x3 y5 z - 1560 x4 y5 z - 192 x5 y5 z +
128 x6 y5 z + 1660 y7 z + 1184 x y7 z - 1464 x2 y7 z - 192 x3 y7 z + 192 x4 y7 z - 472 y9 z -
64 x y9 z + 128 x2 y9 z + 32 y11 z - 752 x4 z2 + 1808 x5 z2 - 1468 x6 z2 + 512 x7 z2 -
64 x8 z2 + 6280 x2 y2 z2 - 5728 x3 y2 z2 - 4066 x4 y2 z2 + 5088 x5 y2 z2 - 820 x6 y2 z2 -
384 x7 y2 z2 + 96 x8 y2 z2 - 136 y4 z2 - 7536 x y4 z2 + 112 x2 y4 z2 + 8640 x3 y4 z2 -
2652 x4 y4 z2 - 1152 x5 y4 z2 + 400 x6 y4 z2 + 2710 y6 z2 + 4064 x y6 z2 - 3100 x2 y6 z2 -
1152 x3 y6 z2 + 624 x4 y6 z2 - 1204 y8 z2 - 384 x y8 z2 + 432 x2 y8 z2 + 112 y10 z2 +
3896 x3 y z3 - 7108 x4 y z3 + 3072 x5 y z3 + 768 x6 y z3 - 768 x7 y z3 + 128 x8 y z3 -
3272 x y3 z3 - 4936 x2 y3 z3 + 8704 x3 y3 z3 - 80 x4 y3 z3 - 2496 x5 y3 z3 + 608 x6 y3 z3 +
2172 y5 z3 + 5632 x y5 z3 - 2464 x2 y5 z3 - 2688 x3 y5 z3 + 1056 x4 y5 z3 - 1616 y7 z3 -
960 x y7 z3 + 800 x2 y7 z3 + 224 y9 z3 + 752 x4 z4 - 1792 x5 z4 + 1472 x6 z4 - 512 x7 z4 +
64 x8 z4 - 3031 x2 y2 z4 + 1936 x3 y2 z4 + 2700 x4 y2 z4 - 2304 x5 y2 z4 + 448 x6 y2 z4 +
697 y4 z4 + 3728 x y4 z4 + 24 x2 y4 z4 - 3072 x3 y4 z4 + 984 x4 y4 z4 - 1204 y6 z4 -
1280 x y6 z4 + 880 x2 y6 z4 + 280 y8 z4 - 800 x3 y z5 + 1488 x4 y z5 - 768 x5 y z5 +
128 x6 y z5 + 992 x y3 z5 + 1016 x2 y3 z5 - 1728 x3 y3 z5 + 480 x4 y3 z5 - 472 y5 z5 -
960 x y5 z5 + 576 x2 y5 z5 + 224 y7 z5 + 16 x4 z6 + 388 x2 y2 z6 - 384 x3 y2 z6 + 96 x4 y2 z6 -
76 y4 z6 - 384 x y4 z6 + 208 x2 y4 z6 + 112 y6 z6 - 64 x y3 z7 + 32 x2 y3 z7 + 32 y5 z7 + 4 y4 z8

```

```

ContourPlot3D[cartesianKlein[x, y, z] == 0, {x, -5, 5},
{y, -5, 5}, {z, -5, 5}, Mesh -> False, Boxed -> False, Axes -> False]

```

